GCSE Maths – Algebra

Expanding Brackets

Worksheet

WORKED SOLUTIONS

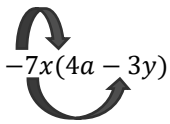
This worksheet will show you how to work out different types of expanding brackets questions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

Expand the expression $-7x(4a-3y)$

Step 1: Draw arrows from the terms outside the bracket to each of the terms inside the bracket, to show which multiplications need to be carried out.



Step 2: Compute required multiplications and sum them together.

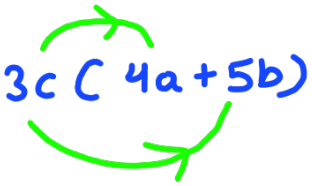
$$-7x(4a-3y) = [-7x \times 4a] + [-7x \times -3y] = \mathbf{-28ax + 21xy}$$

When two or more letters are multiplied, they are written alphabetically next to each other in the final answer.

Guided Example

Expand the expression $3c(4a+5b)$

Step 1: Draw arrows from the terms outside the bracket to each of the terms inside the bracket, to show which multiplications need to be carried out.



Step 2: Compute required multiplications and sum them together.

$3c(4a+5b) = [3c \times 4a] + [3c \times 5b]$

$= 12ac + 15bc$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

1. Expand the following expressions:

a) $4a(2b + 2)$

$4a(2b+2) = 4a(2b) + 4a(2)$
$= 8ab + 8a$

b) $6p(7q - 8)$

$6p(7q-8) = 6p(7q) + 6p(-8)$
$= 42pq - 48p$

c) $-3a(-10d + 6a)$

$-3a(-10d+6a) = -3a(-10d) + (-3a)(6a)$
$= 30ad - 18a^2$

d) $8x(-2x + 3y)$

$8x(-2x+3y) = 8x(-2x) + 8x(3y)$
$= -16x^2 + 24xy$

e) $5ab(2a - 6b^2)$

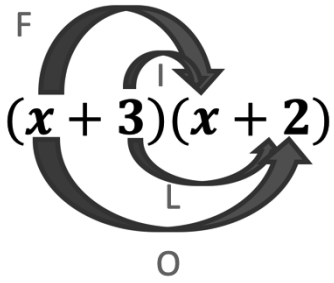
$5ab(2a-6b^2) = 5ab(2a) + 5ab(-6b^2)$
$= 10a^2b - 30ab^3$

Section B

Worked Example

Expand the binomial expression $(x+3)(x+2)$

Step 1: Expand the expression using the FOIL method. You can draw arrows to ensure that you have multiplied all the terms in the first bracket with all the terms in the second bracket.



Step 2: Write each multiplication and sum them together.

F: $x \times x = x^2$
O: $x \times +2 = +2x$
I: $+3 \times x = +3x$
L: $+3 \times +2 = +6$

$$(x+3)(x+2) = x^2 + 2x + 3x + 6$$

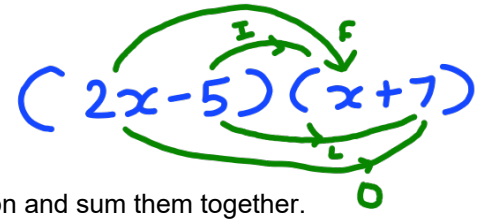
Step 3: Collect any like terms.

$$x^2 + 2x + 3x + 6 = \boldsymbol{x^2 + 5x + 6}$$

Guided Example

Expand the binomial expression $(2x-5)(x+7)$

Step 1: Expand the expression using the FOIL method. You can draw arrows to ensure that you have multiplied all the terms in the first bracket with all the terms in the second bracket.



Step 2: Write each multiplication and sum them together.

F: $2x \times x = 2x^2$
O: $2x \times 7 = 14x$
I: $-5 \times x = -5x$
L: $-5 \times 7 = -35$

$= 2x^2 + 14x - 5x - 35$

Step 3: Collect any like terms

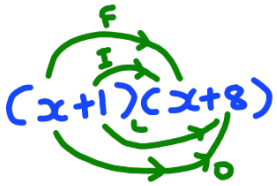
$2x^2 + 14x - 5x - 35 = 2x^2 + 9x - 35$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

2. Expand the following expressions:

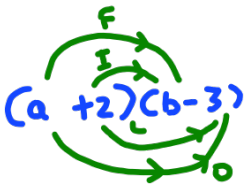
a) $(x+1)(x+8)$

$(x+1)(x+8) = x(x+8) + 1(x+8)$
$= x^2 + 8x + x + 8$
$= x^2 + 9x + 8$



b) $(a+2)(b-3)$

$(a+2)(b-3) = a(b-3) + 2(b-3)$
$= ab - 3a + 2b - 6$

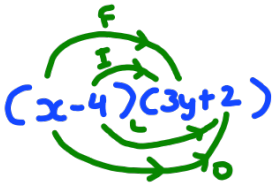


c) $(2p+5)(p+1)$

$(2p+5)(p+1) = 2p(p+1) + 5(p+1)$
$= 2p^2 + 2p + 5p + 5$
$= 2p^2 + 7p + 5$

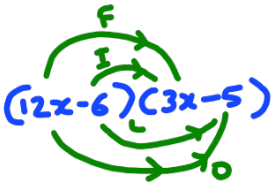
d) $(x-4)(3y+2)$

$(x-4)(3y+2) = x(3y+2) - 4(3y+2)$
$= 3xy + 2x - 12y - 8$



e) $3(4x-2)(3x-5)$

$3(4x-2) = 12x-6$
$3(4x-2)(3x-5) = (12x-6)(3x-5)$
$= 12x(3x-5) - 6(3x-5)$
$= 36x^2 - 60 - 18x + 30 = 36x^2 - 78x + 30$



f) $-2(7y-9)(2z+5)$

$-2(7y-9) = -14y + 18$
$-2(7y-9)(2z+5) = (-14y+18)(2z+5)$
$= -14y(2z+5) + 18(2z+5)$
$= -28yz - 70y + 36z + 90$

Section C

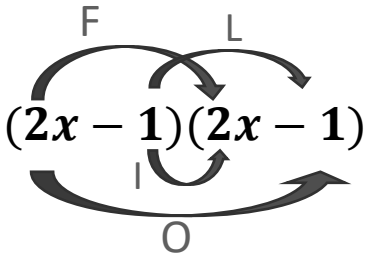
Worked Example

Expand the expression $(2x-1)^2$

Step 1: Write the expression as the product of two brackets

$$(2x-1)^2 = (2x-1)(2x-1)$$

Step 2: Expand the expression using the FOIL method. You can draw arrows to ensure that you have multiplied all the terms in the first bracket with all the terms in the second bracket.



F: $2x \times 2x = 4x^2$
O: $2x \times -1 = -2x$
I: $-1 \times 2x = -2x$
L: $-1 \times -1 = +1$

$$(2x-1)(2x-1) = 4x^2 - 2x - 2x + 1$$

Step 3: Collect any like terms.

$$(2x-1)(2x-1) = \mathbf{4x^2 - 4x + 1}$$

Guided Example

Expand the expression $(4x+7)^2$

Step 1: Write the expression as the product of two brackets

$(4x+7)^2 = (4x+7)(4x+7)$

Step 2: Expand the expression using the FOIL method. You can draw arrows to ensure that you have multiplied all the terms in the first bracket with all the terms in the second bracket.

F : $4x \times 4x = 16x^2$
O : $4x \times 7 = 28x$
I : $7 \times 4x = 28x$
L : $7 \times 7 = 49$

$= 16x^2 + 28x + 28x + 49$

Step 3: Collect any like terms.

$16x^2 + 28x + 28x + 49$
$= 16x^2 + 56x + 49$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

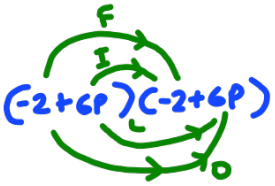
3. Expand the following expressions:

a) $(5a+4)^2$

$(5a+4)^2 = (5a+4)(5a+4)$
$= 5a(5a+4) + 4(5a+4)$
$= 25a^2 + 20a + 20a + 16$
$= 25a^2 + 40a + 16$

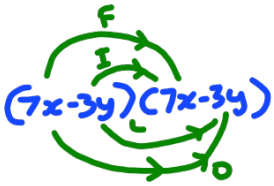
b) $(-2+6p)^2$

$(-2+6p)^2 = (-2+6p)(-2+6p)$
$= -2(-2+6p) + 6p(-2+6p)$
$= 4 - 12p - 12p + 36p^2$
$= 4 - 24p + 36p^2$



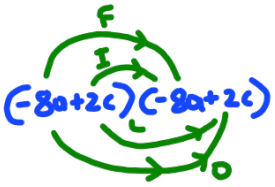
c) $(7x-3y)^2$

$(7x-3y)^2 = (7x-3y)(7x-3y)$
$= 7x(7x-3y) - 3y(7x-3y)$
$= 49x^2 - 21xy - 21xy + 9y^2$
$= 49x^2 - 42xy + 9y^2$



d) $4(-8a+2c)^2$

$(-8a+2c)^2 = (-8a+2c)(-8a+2c)$
$= -8a(-8a+2c) + 2c(-8a+2c)$
$= 64a^2 - 16ac - 16ac + 4c^2$
$= 64a^2 - 32ac + 4c^2$
$4(-8a+2c)^2 = 4(64a^2 - 32ac + 4c^2)$
$= 256a^2 - 128ac + 16c^2$



Section D

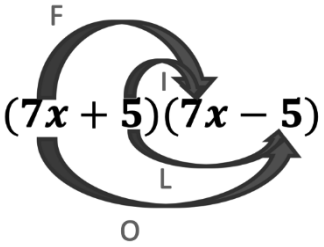
Worked Example

Expand the expression $(7x + 5)(7x - 5)$

Step 1: Expand the expression using the FOIL method. You can draw arrows to ensure that you have multiplied all the terms in the first bracket with all the terms in the second bracket.

F: $7x \times 7x = 49x^2$
O: $7x \times -5 = -35x$
I: $+5 \times 7x = +35x$
L: $+5 \times -5 = -25$

$(7x + 5)(7x - 5) = 49x^2 - 35x + 35x - 25$



Step 2: Collect any like terms.

$$(7x + 5)(7x - 5) = \mathbf{49x^2 - 25}$$

Step 3: Check your answer using the difference of two squares formula.

The expression in the question follows the general form of $(a + b)(a - b)$. *This is known as the* ***difference of two squares****. In this example* $a = 7x$ *and* $b = 5$.

The formula $(a + b)(a - b) = a^2 - b^2$ *means that we know the answer must be*

$$(7x)^2 - 5^2 = 49x^2 - 25.$$

It is important that you can carry out the expansion yourself, as well as being able to use the formula.

Guided Example

Expand the expression $(3y + 5)(3y - 5)$

Step 1: Expand the expression using the FOIL method. You can draw arrows to ensure that you have multiplied all the terms in the first bracket with all the terms in the second bracket.

$(3y+5)(3y-5)$
$= 3y(3y-5) + 5(3y-5)$
$= 9y^2 - 15y + 15y - 25$

Step 2: Collect any like terms.

$= 9y^2 - 25$

Step 3: Check your answer using the difference of two squares formula.

$(3y+5)(3y-5) = (3y)^2 - 5^2 = 9y^2 - 25$ ✓

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

4. Expand the following expressions:

$(a+b)(a-b)$
a^2-b^2 ⇒ can be used for all expansions

a) $(a+9)(a-9)$

$(a+9)(a-9) = a(a-9) + 9(a-9)$
$= a^2 - 9a + 9a - 81$
$= a^2 - 81$
$= a^2 - 9^2$

b) $(d-3)(d+3)$

$(d-3)(d+3) = d(d+3) - 3(d+3)$
$= d^2 + 3d - 3d - 9$
$= d^2 - 9$
$= d^2 - 3^2$

c) $(3z-6)(3z+6)$

$(3z-6)(3z+6) = 3z(3z+6) - 6(3z+6)$
$= 9z^2 + 18z - 18z - 36$
$= 9z^2 - 36$
$= (3z)^2 - 6^2$

d) $5(7-8x)(7+8x)$

$(7-8x)(7+8x) = 7(7+8x) - 8x(7+8x)$
$= 49 + 56x - 56x - 64x^2$
$= 49 - 64x^2 = 7^2 - (8x)^2$

$5(7-8x)(7+8x) = 5(49-64x^2)$
$= 245 - 320x^2$

e) $-6(3x+8y)(3x-8y)$

$(3x+8y)(3x-8y) = 3x(3x-8y) + 8y(3x-8y)$
$= 9x^2 - 24xy + 24xy - 64y^2$
$= 9x^2 - 64y^2 = (3x)^2 - (8y)^2$

$-6(3x+8y)(3x-8y) = -6(9x^2 - 64y^2)$
$= -54x^2 + 384y^2$

Section E

Worked Example

Expand the expression $(3x+1)(5x-2)(8x+4)$

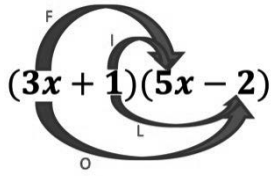
Step 1: Choose two of the brackets to expand first.

For simplicity we will pick the first two brackets. We will group these two together but keep the remaining third bracket there, so we don't forget about it:

$$(3x+1)(5x-2)(8x+4) = [(3x+1)(5x-2)]\,(8x+4)$$

Step 2: Expand these two brackets using the FOIL method, then add up the terms and collect like terms

F: $3x \times 5x = 15x^2$
O: $3x \times -2 = -6x$
I: $+1 \times 5x = +5x$
L: $+1 \times -2 = -2$



$$(3x+1)(5x-2) = 15x^2 - 6x + 5x - 2 = 15x^2 - x - 2$$

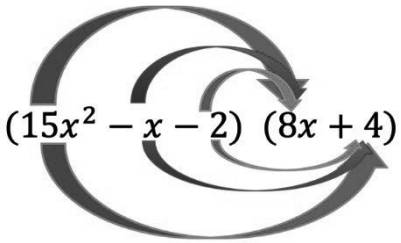
Step 3: Multiply the answer from the expansion of the two brackets with the remaining bracket

Multiply $15x^2 - x - 2$ *with the remaining bracket* $(8x+4)$

$$(3x+1)(5x-2)(8x+4) = (15x^2 - x - 2)(8x+4)$$

*We **CANNOT** use the FOIL method as there are three terms in the first bracket. However, we can expand it by multiplying each term in the first bracket with each term in the second bracket.*

RED: $15x^2 \times 8x = 120x^3$
RED: $15x^2 \times +4 = +60x^2$
BLUE: $-x \times 8x = -8x^2$
BLUE: $-x \times +4 = -4x$
GREEN: $-2 \times 8x = -16x$
GREEN: $-2 \times +4 = -8$



*It is colour-coded to make it clear that every term in the first bracket is multiplied with every term in the second bracket. Now write **each multiplication** and **add them up**.*

$$(3x+1)(5x-2)(8x+4) = (15x^2 - x - 2)(8x+4) = 120x^3 + 60x^2 - 8x^2 - 4x - 16x - 8$$

Step 4: Collect any like terms.

$$(3x+1)(5x-2)(8x+4) = 120x^3 + 60x^2 - 8x^2 - 4x - 16x - 8 = 120x^3 + 52x^2 - 20x - 8$$

So, the final answer is $\mathbf{120x^3 + 52x^2 - 20x - 8}$.

Guided Example

Expand the expression $(7x + 2)(5x + 4)(9x + 2)$

Step 1: Choose two of the brackets to expand first.

$(7x+2)(5x+4)$

Step 2: Expand these two brackets using the FOIL method, then add up the terms and collect like terms.

$(7x+2)(5x+4)$

$= 7x(5x+4) + 2(5x+4)$

$= 35x^2 + 28x + 10x + 8$

$= 35x^2 + 38x + 8$

Step 3: Multiply the answer from the expansion of the two brackets with the remaining bracket.

*We **CANNOT** use the FOIL method as there are three terms in the first bracket. However, we can expand it by multiplying each term in the first bracket with each term in the second bracket.*

$(35x^2 + 38x + 8)(9x+2)$

$= 35x^2(9x+2) + 38x(9x+2) + 8(9x+2)$

$= 315x^3 + 70x^2 + 342x^2 + 76x + 72x + 16$

Step 4: Collect any like terms.

$= 315x^3 + 70x^2 + 342x^2 + 76x + 72x + 16$

$= 315x^3 + 412x^2 + 148x + 16$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

5. Expand the following expressions:

a) $(x+1)(x+3)(x-4)$

$(x+1)(x+3) = x(x+3)+1(x+3)$
$= x^2+3x+x+3$
$= x^2+4x+3$

$(x+1)(x+3)(x-4) = (x^2+4x+3)(x-4)$
$= x^2(x-4)+4x(x-4)+3(x-4)$
$= x^3-4x^2+4x^2-16x+3x-12$
$= x^3-13x-12$

b) $(5y+2)(y-9)(2y+8)$

$(5y+2)(y-9) = 5y(y-9)+2(y-9)$
$= 5y^2-45y+2y-18$
$= 5y^2-43y-18$

$(5y+2)(y-9)(2y+8) = (5y^2-43y-18)(2y+8)$
$= 5y^2(2y+8)-43y(2y+8)-18(2y+8)$
$= 10y^3+40y^2-86y^2-344y-36y-144$
$= 10y^3-46y^2-380y-144$

c) $(2x+7)(4x+1)^2$

$(4x+1)^2 = (4x+1)(4x+1)$
$= 4x(4x+1)+1(4x+1)$
$= 16x^2+4x+4x+1$
$= 16x^2+8x+1$

$(2x+7)(4x+1)^2 = (2x+7)(16x^2+8x+1)$
$= 2x(16x^2+8x+1)+7(16x^2+8x+1)$
$= 32x^3+16x^2+2x+112x^2+56x+7$
$= 32x^3+128x^2+58x+7$

d) $(7x+1)(7x-1)(6x+8)$

$(a+b)(a-b) = a^2 - b^2$

$(7x+1)(7x-1) = (7x)^2 - 1^2 = 49x^2 - 1$

$(7x+1)(7x-1)(6x+8) = (49x^2-1)(6x+8)$

$= 49x^2(6x+8) - 1(6x+8)$

$= 294x^3 + 392x^2 - 6x - 8$

e) $(8x+6)^3$ $= (8x+6)(8x+6)(8x+6)$

$(8x+6)(8x+6) = 8x(8x+6) + 6(8x+6)$

$= 64x^2 + 48x + 48x + 36$

$= 64x^2 + 96x + 36$

$(8x+6)(8x+6)(8x+6) = (64x^2 + 96x + 36)(8x+6)$

$= 64x^2(8x+6) + 96x(8x+6) + 36(8x+6)$

$= 512x^3 + 384x^2 + 768x^2 + 576x + 288x + 216$

$= 512x^3 + 1152x^2 + 864x + 216$

f) $(-2+3x)(2y+1)(4x-7)$

$(-2+3x)(2y+1) = -2(2y+1) + 3x(2y+1)$

$= -4y - 2 + 6xy + 3x$

$(-2x+3)(2y+1)(4x-7) = (-4y - 2 + 6xy + 3x)(4x-7)$

$= 4x(-4y - 2 + 6xy + 3x) - 7(-4y - 2 + 6xy + 3x)$

$= -16xy - 8x + 24x^2y + 12x^2$

$+ 28y + 14 - 42xy - 21x$

$= -58xy - 29x + 24x^2y + 12x^2 + 28y + 14$